A Relativistic Look
at Quantum Field
Theory

Oxford
May 1986

CLASSICAL CONCEPT OF FIELD

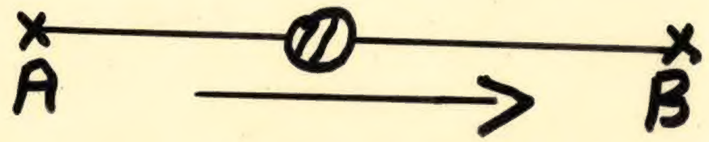
FIELD THEORY: Associates contain properties with Space-Time Points

Compare

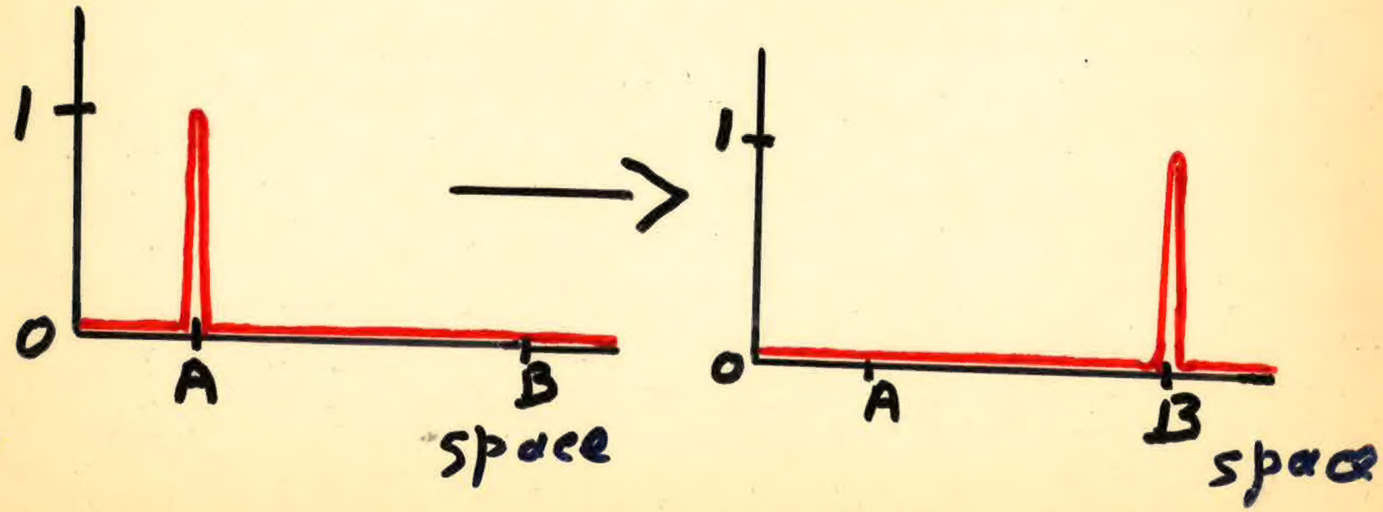
PARTICLE THEORY: Attributes to contain individuals (the particles) a variety of properties

MOTION FROM A TO B

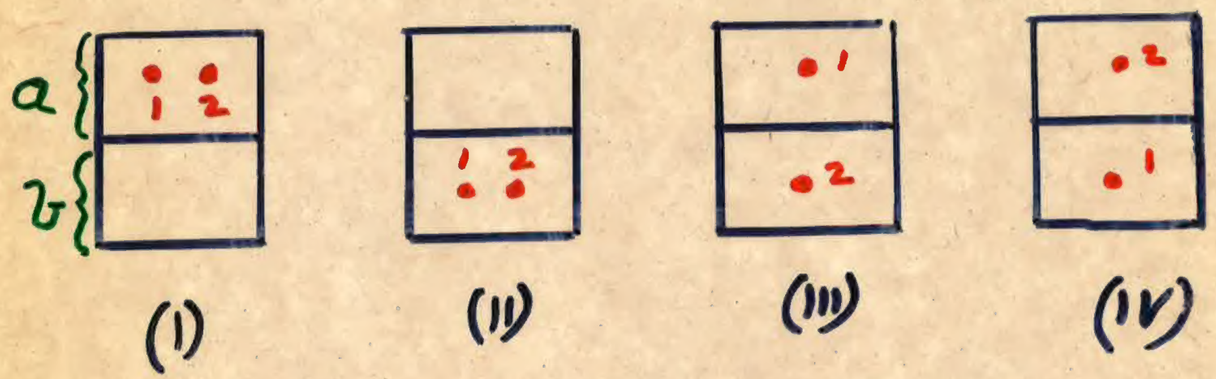
Particle Description



Field Description



Statistical weights for 2-particle System



In Quantum Stat. Mech. (III) and (IV) are regarded as one and the same state for the purposes of assigning Statistical weights

Quantum Statistical Mechanics

Consider the 4 product wave functions

$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{I}$$
$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2 \quad \text{II}$$
$$\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{III}$$
$$\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1) \quad \text{IV}$$

4-dimensional vector space equally well spanned by

Symmetric
$$\psi_a(\underline{r}_1)\cdot\psi_a(\underline{r}_2) \quad \text{V}$$
$$\psi_b(\underline{r}_1)\cdot\psi_b(\underline{r}_2) \quad \text{VI}$$
$$1/\sqrt{2}\,(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)+\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)) \quad \text{VII}$$

Antisymmetric
$$1/\sqrt{2}\,(\psi_a(\underline{r}_1)\cdot\psi_b(\underline{r}_2)-\psi_a(\underline{r}_2)\cdot\psi_b(\underline{r}_1)) \quad \text{VIII}$$

THE INDISTINGUISHABILITY PRINCIPLE (IP)

Two particles are indistinguishable if $\langle P\phi|Q|P\phi\rangle = \langle\phi|Q|\phi\rangle$

$\forall Q, P, \phi$

IP can be taken as a restriction

on observables $\Rightarrow$ P commutes with Q, i.e. Q is a symmetric function of particle labels $\rightarrow$ parastatistics

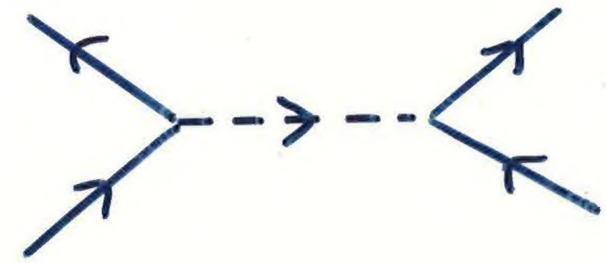
or

on states $\Rightarrow$ $P|\phi\rangle = \pm|\phi\rangle$

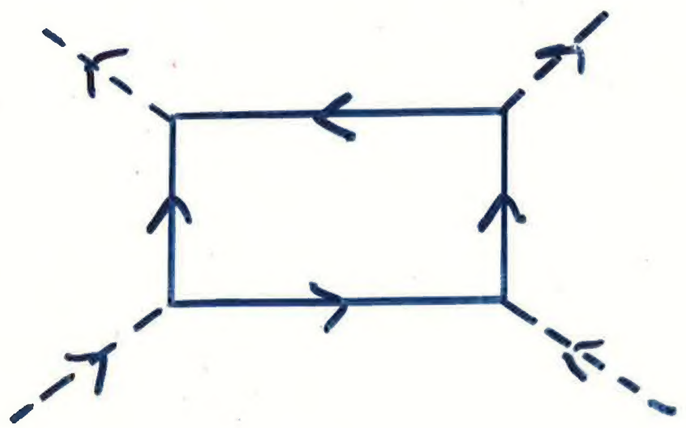
So Bosons and Fermions only allowed

MATTER AND FORCE

Compare



with



So Which is the 'matter' particle and which is the 'force' particle?

A Philosopher Looks at QFT Contd.

But 2nd quantization is more general than the N-Particle Schrödinger Eq. because of the constraint

$$\sum_i n_i = N$$

FOCK SPACE

$$\mathcal{F} = K_0 \oplus K_1 \oplus \cdots K_N \oplus \cdots$$

↓ vacuum

CREATION and ANNIHILATION OPERATORS

$$\left.\begin{aligned} a_i \,|n_i\rangle &= \sqrt{n_i}\cdot|n_i-1\rangle \\ a_i^+ \,|n_i\rangle &= \sqrt{n_i+1}\cdot|n_i+1\rangle \end{aligned}\right\}$$